Team Control Number

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Problem Chosen

B

2013 Mathematical Contest in Modeling (MCM) Summary Sheet

(Attach a copy of this page to your solution paper.)

China is faced with a serious fresh water crisis, namely, severe water quantity shortage and serious water pollution. In this paper, we attempt to find the best water strategy for 2013 to meet the projected water needs of China in 2025.

Grey Prediction model is introduced to forecast the water consumption of each province in China from 2013 to 2025 based on the statistics obtained from the Statistical Yearbook of China. Error analysis is basically applied to assess the precision of our prediction.

In the mean time, Fuzzy Mathematical model is employed to classify the degrees of water shortage of 31 provinces. The provinces are divided into 3 categories according to the degrees: provinces that have adequate water, provinces which are short of water, and the remaining provinces which balance water supply with consumption.

Minimum Spanning Tree (MST) model is utilized to optimize the plan of water transformation. We decide to adopt a method that we build the watercourses, letting water flow through provinces with enough water and provinces which lack water. MST model is applied to find the shortest and the most economical watercourses.

The next model called Local Dynamic Programming (LDP) is used to find the period of water delivery, which is devoted to the economical impact of water storage and movement, de-salinition and conservation. Different districts can determine their inputs rely on their actual situation, and use our model to acquire the relative water strategy.

Finally, we provide the Monte Carlo method to testify the water strategy, which verifies the rationality of our prediction in turn.

In a nutshell, we use four models above, which rely on the actual water demand in outputting the optimal period of water delivery to reach the minimum cost of water storage and movement, de-salinition and conservation.

Key words: Water strategy, Grey prediction model, Fuzzy mathematical model, Minimum spanning tree model, Monte Carlo theory

Best Water Strategy

Contents

1 Introduction

We live on a blue planet. The ocean covers 71 percents of the Earth's surface[1], however, most of that water on Earth is so salty that we cannot use directly, only about 2.5 percents of the planet´s total surface water is fresh[2], most of which is frozen ice or deep groundwater. Merely 0.01 percent or so can be used by human directly[3], it is estimated that one-fifth of the worlds population[4][5] live under the circumstance that still lack access to safe water and over 2.5 billion lack access to adequate sanitation[6]. At the beginning of the 21th century, scientists and experts pointed out that the future adequacy of freshwater resources is difficult to assess, owing to the climate change and population growth[4]. Because of these two main factors, nations should firstly make spacial assessment of the global-scale consequences of their local water management practices[7], then find a more advanced way to solve the problem of the water.

Although possessing the vast precise resources, China is faced with the problems of both severe water quantity shortages and severe water quality pollution[8]. The amount of water available per person in China is just a quarter of the world average, ranking 88th in 153 countries, which ranked by the World Bank[9], and the amount of water available per person of 16 provinces is below the line of severe water shortage. Furthermore, the distribution of water resources is uneven between north and south, east and west. In the meantime, we cannot ignore the high number of river pollution incidents occurred in China, such as the high profile Songhua River toxic chemical spill in November 2005, and the pollution of drinking water source by algae in the Tai Lake, Wuxi in May 2007[10]. Faced with the two water problems, the Chinese government launched projects, including the Three Gorges Dam, and the South-to-North Water Diversion Project. And also put forward a plan that China wants to reduce water-use per unit of gross domestic product by 60 percents, in 2009[11][12]. In 2011, the Chinese government has announced that it will invest four trillion renminbi over the next ten years to improve the quality of water[13], which is the years No 1 Document, which has the top priorities. Thus, it can be seen the Chinese government pay much attention to the issues of water resources.

2 Representation of Problem

First of all, we are asked to build the mathematical model for determining an effective, feasible, and cost-efficient water strategy for 2013 to meet the demand of projected water needs of in 2025, and to figure out the best water strategy, which must includes projects of storage, movement, de-salinization and conservation. Next, we are supposed to use our model to discuss the economic, physical and environmental implications of our strategy. Hand in a non-technical position paper, which stresses our approach, its feasibility and costs, to the government, then explain why it is the "best water strategy choice".

3 Assumptions

- We are given accurate data regarding water consumption of 31 provinces and other relavant factors.
- The provinces in China will not exprience mass migration of population.

- The population growth is constant.
- South-to-North Water Diversion Project will be completed on schedule.
- The world is in peace and stability.

4 The Prediction of Water Consumption from 2012 to 2025

We decide to use the grey prediction model to forecast the future values of water consumption from 2012 to 2025.

4.1 Grey Prediction Model

- Terminology:
  $X^{(0)}$: the original data sequence

  $X^{(0)}(k)$: the item(k) of the original data sequence

  $X''^{(0)}$: the predicted data sequence

  $X''^{(0)}(k)$: the item(k) of the predicted data sequence

  $\varepsilon^{(0)}$: the residual sequence:
  $$\varepsilon^{(0)} = X''^{(0)} - X^{(0)}$$

  $\Delta^{(0)}$: the relative error sequence:
  $$\Delta^{(0)} = \left| \frac{\varepsilon^{(0)}}{X^{(0)}} \right|$$

  η: the grey absolute correlation degree:
  $$\eta = \frac{1 + |S_0| + \left|S_0''\right|}{1 + |S_0| + \left|S_0''\right| + \left|S_0'' - S_0\right|}$$

  Where
  $$|S_0| = \left| \sum_{k=2}^{n-1} X^{(0)}(k) + \frac{1}{2} X^{(0)}(n) \right|$$

  Then
  $$\left|S_0''\right| = \left| \sum_{k=2}^{n-1} X''^{(0)}(k) + \frac{1}{2} X''^{(0)}(n) \right|$$

  And
  $$\left|S_0'' - S_0\right| = \left| \sum_{k=2}^{n-1} [X''^{(0)}(k) - X^{(0)}(k)] + \frac{1}{2} [X''^{(0)}(n) - X^{(0)}(n)] \right|$$

  $\overline{x}$: the mean value of $X^{(0)}$: $\overline{x} = \frac{1}{n} \sum\limits_{k=1}^{n} X^{(0)}(k)$

  S_1^2: the variance of $X^{(0)}$: $S_1^2 = \frac{1}{n} \sum\limits_{k=1}^{n} [X^{(0)}(k) - \overline{x}]^2$

$\overline{\varepsilon}$: the mean value of $\overline{\varepsilon}$: $\overline{\varepsilon} = \frac{1}{n}\sum\limits_{k=1}^{n} \varepsilon^{(0)}(k)$

S_2^2: the variance of S_2^2: $S_2^2 = \frac{1}{n}\sum\limits_{k=1}^{n} \left[\varepsilon^{(0)}(k) - \overline{\varepsilon}\right]^2$

C: the mean square error ratio: $C = \frac{S_2}{S_1}$

- The Background of the Grey Prediction of Model:
The grey prediction theory is a newly arisen cross-cutting academics, which was initiated by the distinguished professor Julong Deng in 1982, and by now over 30 years old[14]. When concerned about the information, the systems which lack information, such as structure message, operation mechanism and behavior document, are referred to as Grey Systems[15], which are distinguished with the White Systems and the Black Systems. For instance, the human body, agriculture, economy, irrigating and so on, are Grey Systems. We use the original data sequence $X^{(0)}$:

$$X^{(0)} = \{X^{(0)}(1), X^{(0)}(2), ..., X^{(0)}(n)\} \tag{1}$$

to do the accumulated generating operation[AGO], then get the AGO formation $X^{(1)}$:

$$X^{(1)} = \{X^{(1)}(1), X^{(1)}(2), ..., X^{(1)}(n)\} \tag{2}$$

where

$$X^{(1)}(1) = X^{(0)}(1), X^{(1)}(k) = \sum_{i=1}^{k} X^{(0)}(i), k = 1, 2, 3, ..., n$$

Then, establish a first order differential equation for $X^{(1)}(k)$ as:

$$\frac{dX^{(1)}(k)}{dk} + aX^{(1)}(k) = b \tag{3}$$

We get the solution of (3)that is:

$$X''^{(1)}(k) = [X^{(0)}(1) - \frac{b}{a}]e^{-a(k-1)} + \frac{b}{a} \tag{4}$$

Where

$$[a, b]^T = (B^T B)^{-1} X_n \tag{5}$$

And

$$B = \begin{pmatrix} -Z^{(1)}(2) & 1 \\ -Z^{(1)}(3) & 1 \\ ... & ... \\ -Z^{(1)}(n) & 1 \end{pmatrix}$$

Where

$$Z^{(1)}(k) = \alpha X^{(1)}(k) + [1 - \alpha]X^{(1)}(k - 1)$$
$$X_n = \{X^{(0)}(2), X^{(0)}(2), ..., X^{(0)}(n)\}^T$$

We get $X''^{(1)}(k)$ from (4). Let $X''^{(0)}$ be the fitted and predicted series:

$$X''^{(0)} = \{X^{(0)}(1), X^{(1)}(2), ..., X^{(0)}(n), ...\}$$

Where $X''^{(1)}(1) = X^{(0)}(1)$

$$X''^{(0)}(k) = (1 - e^{a})[X^{(0)}(1) - \frac{b}{a}]e^{-a(k-1)} \tag{6}$$

Thus, we obtain the GM(1,1) fitted sequence: $X''^{(0)}(1), X''^{(0)}(2), ..., X''^{(0)}(n)$, and the GM(1,1) forecast values:$X''^{(0)}(n+1), X''^{(0)}(n+2), X''^{(0)}(n+3), ...$[16]

4.2 Solution of the Prediction of Usage of Water

We use the Internet to log in the website of National Bureau of Statistics of China, then get the statistical yearbook of China, and have access to use the statistics of the water consumption in different provinces from 2003 to 2012[17](Shown in Appendix A), next we use the algorithm of Grey Prediction Theory(Shown in the Background of the Grey Prediction Model) based on the statistics to get the forecast values from 2013 to 2025(Shown in Appendix B). The figure of the water consumption of each province of China also illustrated in Appendix D. For example, Shanghai. We use the basic statistics to form the original data sequence $X^{(0)}$ (1):

$$X^{(0)}_{shanghai} = \{104.27, 109.0, 118.1, 121.3, 118.6, 120.2, 119.8, 125.2, 126.3, 126.3\}$$

Then according to (2), we obtain the AGO formation $X^{(1)}$:

$$X^{(1)}_{shanghai} = \{104.3, 213.3, 331.4, 452.7, 571.3, 691.5, 811.3, 936.5, 1062.8, 1189.1\}$$

And based on (5), we can acquire the parameters:

$$a = -0.0141$$

$$b = 111.5589$$

Finally, we get the forecast value sequence using (6), that is:

$$X''^{(0)}_{shanghai(2003-2010)} = \{104.27, 113.8341, 115.4538, 117.0964, 118.7624, 120.4521, 122.1659, 123.9041\}$$

$$X''^{(0)}_{shanghai(2011-2018)} = \{125.6669, 127.4549, 129.2683, 131.1075, 132.9729, 134.8648, 136.7836, 138.7297\}$$

$$X''^{(0)}_{shanghai(2019-2025)} = \{140.7035, 142.7054, 144.7358, 146.7951, 148.8836, 151.0019, 153.1504\}$$

We can use excel to draw the broken line graph of the actual water consumption and predicted water consumption into the same figure, it is shown in figure 1.

Next we should do the accuracy evaluation of the forecast water consumption of some provinces(Shown in Appendix D). For instance: Shanghai. We can obtain the mean value of relative error sequence of Shanghai

$$\overline{\Delta} = 0.01491$$

Then get the grey absolute correlation degree based on the Terminology:

$$\eta = 0.9978$$

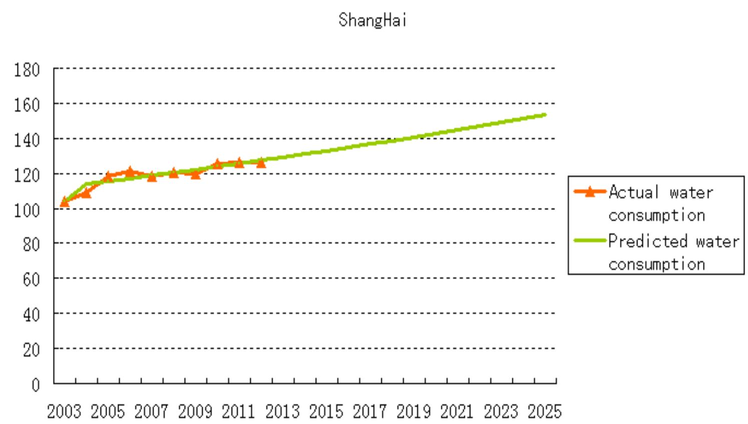

Figure 1: the Water Consumption of Shanghai

Table 1: the Table of Accuracy Evaluation

Degree	Index		
	the mean value of relative error sequence	the grey absolute correlation degree	the mean square error ratio
One	0.01	0.9	0.35
Two	0.05	0.8	0.5
Three	0.1	0.7	0.65
Four	0.2	0.6	0.8

And the mean square error ratio

$$C = 0.3486$$

We construct the table of accuracy evaluation(shown in table 1) to evaluate the accuracy of our prediction.

According to the Appendix C, we can clear see that the mean square error ratio of Zhejiang, Guangxi, Hainan, Yunnan and Ningxia are all over 0.8, which is the upper limit of the fourth degree. Thus, we conclude that Zhejiang, Guangxi, Hainan, Yunnan and Ningxia are not will fitted to our mathematical model. Then, we come up a solution of using the theory of the linear fitting to test whether these five provinces fits well in once fitting, however, the result(Shown in table 2) is frustrating, which is worse than we use the Grey Prediction Theory.

Table 2: The Result of Comparison of two Theory

	Mean square error ratio[GM(1,1)]	Mean square error ratio[linear fitting]
Zhejiang	0.9339	0.9429
Shanxi	0.9094	0.9109
Hainan	0.8137	0.986
Yunnan	0.9031	0.9333
Ningxia	0.8088	0.9807

5 The Classification of the Shortage of Water

We utilize the fuzzy mathematics to get the fuzzy set, and obtain the score of each province, then have the classification of the shortage of water.

5.1 Fuzzy Mathematics

- Terminology:
  U: the set of factors of evaluation

  WRP: the water resources per person

  $GMWR$: the gross amount of water resources

  $RWCG$: the ratio of water consumption and gross amount of water resources

  w: the weight vector

- The Background of The Fuzzy Mathematics:

  In production practice, scientific research, and daily life[18], we usually meet the ambiguous concept or phenomenon, such as big and small, young and old, fast and slow, beauty and ugly, fat and thin and so on. Especially in scientific field, we should do quantitative analysis on all the fuzzy concepts[19]. Thus we need the fuzzy mathematics.

  Fuzzy mathematics forms a branch of mathematics related to fuzzy theory and fuzzy logic[20]. In fuzzy mathematics, we consider the uncertainty involved in the case and then to evaluate the degree of one fuzzy event, and the need to generalize creates fuzzy mathematical objects.

- The Establishment of the Set of Factors:
  In our mathematical model, we establish the set of factors of evaluation, U,

  $$U = (u_1, u_2, u_3, ..., u_n)$$

  Where $u_i (i = 1, 2, 3, ..., n)$ is the influential factor. We directly use water resources per person, gross amount of water resources, and the ratio of water consumption and gross amount of water resources to be the influential factors, that is

  $$U = (WRP, GMWR, RWCG)$$

- Determination of Weight Vector:
  We need to use analytic hierarchy process, known as AHP, to compare the degree of the three factors, and then get the weight vector.

  $$w = (w_1, w_2, w_3, ..., w_n)$$

- The Establishment of Matrix of Evaluation Degree:
  The matrix of evaluation degree is to evaluate the suspicious degree of every node, we set five degrees to describe it , namely,

  $$G = (level1, level2, level3, level4, level5)$$

To make it clear, we use

$$G = (90, 70, 50, 30, 10)$$

from high to low principle.[21]

- Acquirement of Matrix of Fuzzy Evaluation Relationship:
  From the fuzzy mathematical evaluation theory, we can establish the fuzzy sets V based on the trapezoidal distribution.

  If x belongs to the level 1, it should obey

$$\begin{cases} 1, x > 0.9 \\ \frac{x-0.7}{0.9-0.7}, 0.7 < x \leq 0.9 \end{cases}$$

  If x belongs to the level 2, it should obey

$$\begin{cases} 1, x = 0.7 \\ \frac{x-0.5}{0.7-0.5}, 0.5 < x < 0.7 \\ \frac{0.9-x}{0.9-0.7}, 0.7 < x < 0.9 \end{cases}$$

  If x belongs to the level 3, it should obey

$$\begin{cases} 1, x = 0.5 \\ \frac{x-0.3}{0.5-0.3}, 0.3 < x < 0.5 \\ \frac{0.7-x}{0.7-0.5}, 0.5 < x < 0.7 \end{cases}$$

  If x belongs to the level 4, it should obey

$$\begin{cases} 1, x = 0.3 \\ \frac{x-0.1}{0.3-0.1}, 0.1 < x < 0.3 \\ \frac{0.5-x}{0.5-0.3}, 0.3 < x < 0.5 \end{cases}$$

  If x belongs to the level 5, it should obey

$$\begin{cases} 1, x \leq 0.1 \\ \frac{0.3-x}{0.3-0.1}, 0.1 < x < 0.3 \end{cases}$$

  According to the evaluation of fuzzy mathematics, we get

$$R = \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{pmatrix}, 0 \leq r_{ij} \leq 1$$

  where r_{ij} is the administrative relationship between u_i and v_i, thus form the basic of the evaluation of fuzzy mathematics.

- Acquirement of Composition Operator:
  We use the product of the weight vector and matrix of fuzzy evaluation relationship to form the composition operator B, the factors is

$$W = w \cdot R = (w_1, w_2, w_3, ..., w_n) \cdot \begin{pmatrix} r_{11} & \cdots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \cdots & r_{mn} \end{pmatrix} \tag{7}$$

$$W_j = w_1 \cdot r_{1j} + w_2 \cdot r_{2j} + ... + w_n \cdot r_{nj}, j = 1, 2, ..., m \tag{8}$$

5.2 Solution of the Classification of Water Resources of 31 Provinces

Firstly, we use the statistical yearbook of China to get the three influential factors: water resources per person, gross amount of water resources and the ratio of water consumption and gross amount of water resources(Shown in table 3). We assume that

Table 3: Three Influential Factors

Province	WRP	GMWR	RWCG	Province	WRP	GMWR	RWCG
BeiJing	124.2	23.85836984	0.654	HuBei	2216.5	959.4057	2.4241
TianJin	72.8	12.78777778	0.5075	HuNan	2938.7	1593.598	4.8533
HeBei	195.3	140.8031889	0.7712	GuangDong	1943.3	1720.157	3.561
ShanXi	261.5	99.16453942	1.3081	GuangXi	3852.9	1714.896	5.1638
NeiMengGu	1576.1	410.4757144	1.9751	HaiNan	5538.7	349.4529	8.2646
LiaoNing	1392.1	304.9235272	1.7179	ChongQing	1616.8	523.8066	3.4692
JiLin	2503.3	393.5744444	2.4317	SiChuan	3173.5	2416.568	9.0634
HeiLongJiang	2228.6	708.6179222	1.3521	GuiZhou	2726.8	915.962	7.8165
ShangHai	163.1	29.20118989	0.1907	YunNan	4233.1	1881.337	12.0407
JiangSu	489.2	427.1536376	0.5821	XiZang	153681.9	4437.434	83.506
ZheJiang	2608.7	887.8108785	4.5411	ShaanXi	1360.3	428.8392	4.1645
AnHui	1526.9	728.1488889	1.0809	GanSu	841.7	217.3402	1.7992
FuJian	4491.7	1097.958791	4.4196	QingHai	13225	708.4033	21.4633
JiangXi	5116.7	1387.424489	3.5084	NingXia	148.2	9.703889	0.1265
ShanDong	324.4	345.7638113	1.482	XinJiang	5125.2	902.6563	1.4695
HeNan	566.2	445.8613191	1.4268				

the RWCG has the strongest influence among three influential factors, and GMWR is stronger moderately than that of WRP, thus we obtain the pairwise comparison matrix is:

$$\begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ 2 & 1 & \frac{1}{2} \\ 3 & 2 & 1 \end{pmatrix}$$

Based on the pairwise comparison matrix, we can easily get the eigenvector and the maximum eigenvalue of the matrix. and the maximum eigenvalue is

$$\lambda_{\max} = 3.0092$$

The eigenvector of it is:

$$(0.1634, 0.2970, 0.5396)$$

Then, we need to do the consistency check.

$$CI = \frac{\lambda_{\max} - 3}{2}$$

where CI is the index of the consistency check. Because we use three influential factors, thus RI=0.58[19], then we get the consistency ratio CR:

$$CR = \frac{CI}{RI}$$

Next

$$CR = 0.007931 < 0.1$$

We consider the pairwise comparison matrix pass the consistency check.

What is next, we ought to calculate the score of the 31 provinces based on fuzzy mathematics. According to (7)(8) and $score = \sum G \cdot W$, we acquire the table of scores of 31 provinces(Shown in table 4):

Table 4: the table of scores of 31 provinces

Province	Score	Province	Score	Province	Score
XiZang	90	HuBei	24.2203	HeNan	10.0402
SiChuan	54.3404	ZheJiang	23.9567	BeiJing	10
YunNan	47.1813	HaiNan	23.072	TianJin	10
GuangXi	42.9921	HeiLongJiang	19.5269	HeBei	10
JiangXi	40.3148	AnHui	17.9535	ShanXi	10
HuNan	38.1751	JiLin	15.2925	ShangHai	10
GuangDong	37.8071	ChongQing	14.3488	JiangSu	10
FuJian	33.129	NeiMengGu	12.7271	ShanDong	10
XinJiang	31.1993	LiaoNing	12.218	NingXia	10
QingHai	28.0615	ShaanXi	12.1298		
GuiZhou	24.8139	GanSu	10.6948		

We presume that provinces, which scores are above 25, have enough water, they are Xizang, Sichuan, Yunnan, Guangxi, Jiangxi, Hunan, Guangdong, Fujian, Xinjiang, and Qinghai. Scores below 15 are provinces short of water, they are Chongqin, Neimenggu, Liaoning, Shannxi, Gansu, Henan, Beijing, Tianjin, Hebei, Shanxi, Shanghai, Jiangsu, Shandong, and Ningxia. The water resources of the rest of provinces in the mainland are normal level, such as: Guizhou, Hubei, Zhejiang, Hainan, Heilongjiang, Anhui, Jilin.

6 The Determination of the Watercourses

Because we want to find the shortest and the most economical water route between provinces, and also the minimum spanning tree theory usually solve the problem of the shortest distances. Thus, we decide minimum spanning tree to determine the watercourses.

6.1 Minimum Spanning Tree

A: the point set

E: the frontier set

G: the graph

T: the minimum spanning tree of the graph

A spanning tree is the mathematical field of graph theory. The graph in the graph theory is not the basic geometry, but the abstracts of the entities in objective world[19]. A weighed graph$G(V, E)$, which is fully connected, may have many spanning trees. Assuming that $T(V, E)$ is one of the spanning trees in weighed graph G, when we add the weight of each side together, the sum can be called the weight of the spanning tree T. If we can figure out the smallest weigh of all trees, then that can be called the minimum spanning tree.

6.2 Solution of the Watercourses

Through the website of South-to-North Water Diversion(Shown in Figure 2), we get the knowledge that the river routes of Shandong, Tianjin, Jiangsu, Hebei, Beijing and Henan will be constructed before 2025[22].

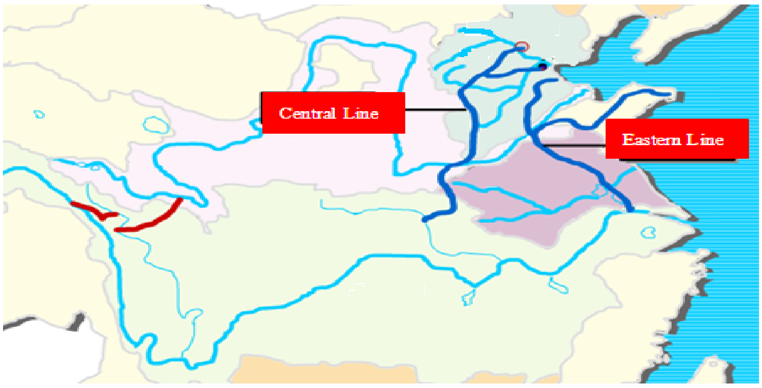

Figure 2: The Sketch Map of South-to-North Water Diversion

First of all, consider provinces are substituted by their capitals, and use numbers to replace provinces, for instance, number one equals Xinjiang. We can see distances between each province(Shown in figure 3), because the watercourse has already constructed in Shandong, Tianjin, Jiangsu, Hebei, Beijing and Henan, then we can view them as one point(No. 15), that is, the red circle we use in figure(3,4).

Table 5: Numbers for Provinces

Number	Province	Number	Province	Number	Province	Number	Province
1	XingJiang	7	ChongQing	13	HuNan	15	BeiJing
2	XiZang	8	ShaanXi	14	GuangDong		TianJin
3	QingHai	9	GanSu	16	JiangXi		JiangSu
4	SiChuan	10	NingXia	17	FuJian		HeBei
5	YunNan	11	NeiMengGu	18	ShangHai		ShanDong
6	GuangXi	12	ShanXi	19	LiaoNing		HeNan

To overcome the shortage of water resources of some provinces, we conceive that using the water of provinces which own enough water to help those lack of water, then we come up a idea, which uses Kruskal algorithm to find the shortest and economical watercourse(Shown in figure 4) to solve the problem.

We can see from the figure that each province have the only way, namely the watercourse, to pass through, which indicates that it is the shortest and economical watercourse.

So far, we have finished the design of the route of water.

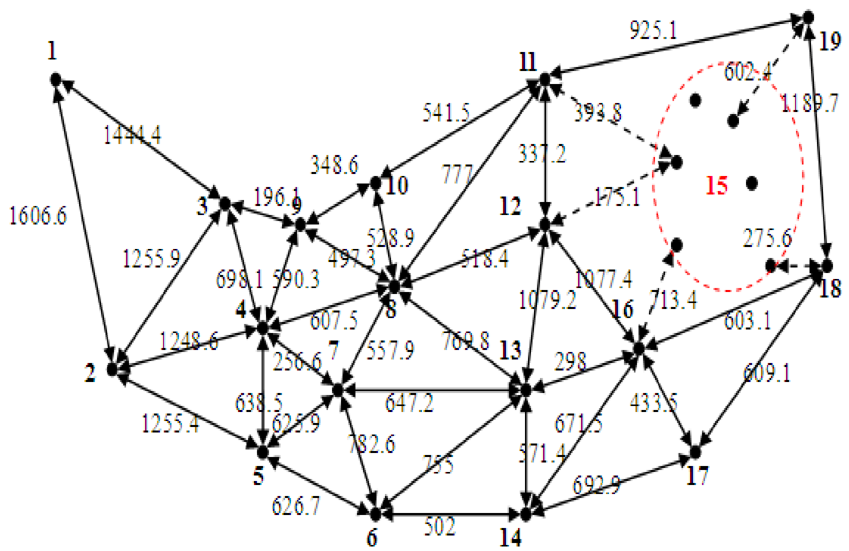

Figure 3: The Distances between Provinces

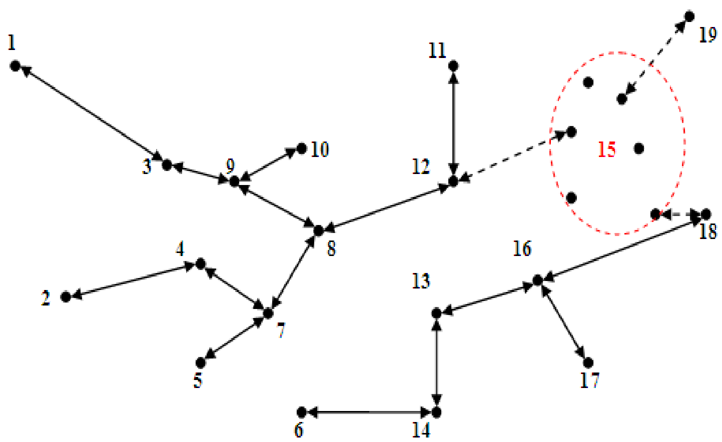

Figure 4: The Solution of Watercourses

7 Storage and Movement, De-salinization and Conservation

7.1 Terminology

$Cost$: the cost per day
s: the fees of storage of fresh water per cubic meter per day
$d1$: the fees of water delivery each time
$d2$: the fees of salinization
$d3$: the fees for the decline of the quality of water due to pollution
$d4$: the fees of management of water
r: the demand of water per cubic meter per day
q: the amount of water delivering
qb: the buffer stock of water
t: the time(day)
nw: the amount of water needs per day
NW: the amount of water needs per cycle
T: the cycle of delivery of water(day)

7.2 The Water Strategy

7.2.1 the Introduction of the Idea

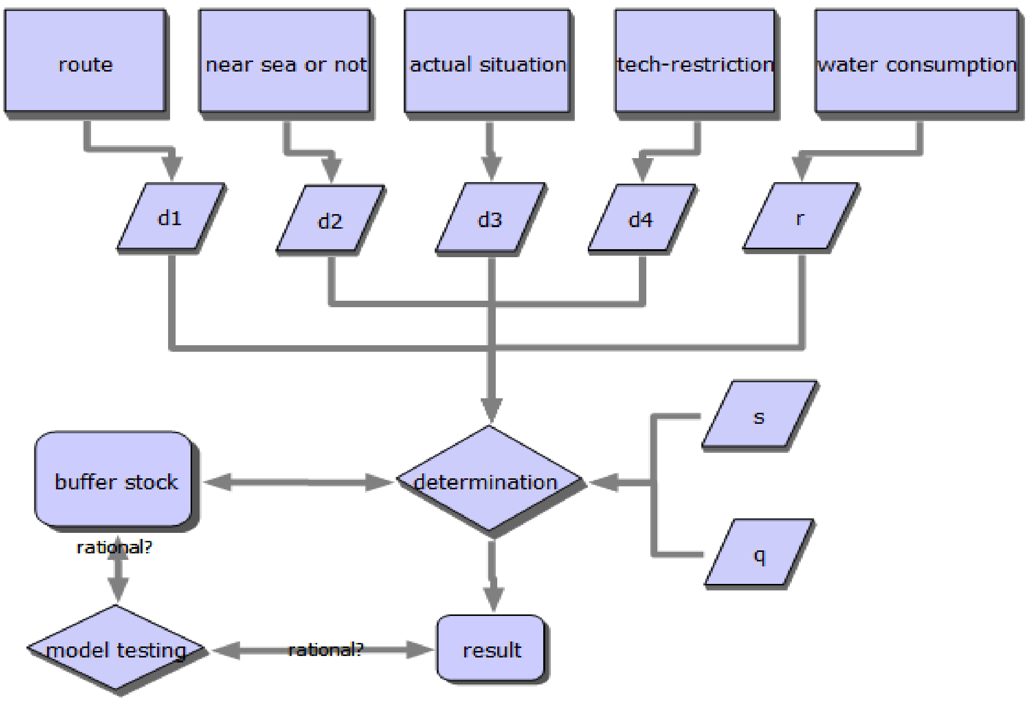

Figure 5: The Flow Chart of Our Solution

We hope that we can make the sum of d1, d2, d3, d4 and s be as low as possible, namely, make the total cost be lowest, including administrative cost and compensation of employees. We presume that the need of water and cost sustain to be a constant during short period, thus we should focus on the question below[23]:

Provided that the storage of fresh water in each province can meet the need, we focus our attention on the amount of water delivering every day, desalination, the cost of storage, and the fees used for the decline of the quality of water due to pollution, and make them as low as possible.

Then we need also consider the metamorphic problem of storage of fresh water, the stability of necessity of water. In our model, we just consider the following variables: s, d1, d2, d3, d4, and r. That is:

$$Cost = f(s, d1, d2, d3, d4, r)$$

7.2.2 Strategic Model

- The sub-model of storage:

  We need to consider the fees of storage of water change over the amount of storage, in our model we consider s to be the constant.

  Requirement:

  In a particular area, if we draw the daily demand of water, we will find that the demand spread compactly around the frequency of maximum demand, then we can think that the water demand r is constant(Shown in figure 6). The same demand happens in the discrete case, but we think that it is continuous.

  The buffer stock of water:

  Because the demand fluctuates in reality, so we must put the buffer stock of water into consideration(Shown in Figure 7).

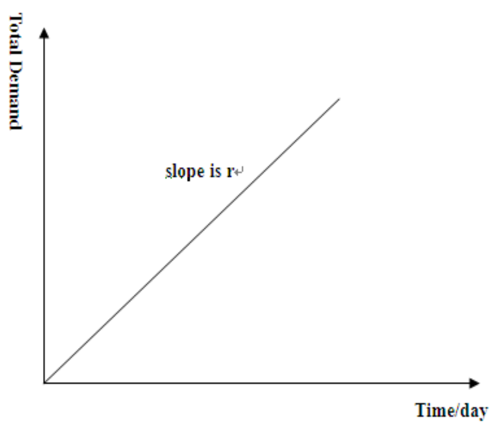

Figure 6: Constant of r

- Solution of the model:

The sum of d1, d2, d3, d4 is

$$d = d1 + d2 + d3 + d4$$

The amount of water delivering equals to the demands:

$$q = r \cdot t$$

The total fee of every cycle F_{cycle} is:

$$F_{cycle} = d + s \cdot q/2 \cdot t + qb \cdot t \cdot s$$

The fee of daily use

$$F_{daily} = \frac{d}{t} + \frac{s \cdot q}{2} + qb \cdot s$$

Thus,

$$Cost = \frac{d}{t} + \frac{s \cdot r \cdot t}{2}$$

Because we want to obtain the extremum of the Cost, we do the differential operation, and then we obtain:

$$\frac{dCost}{dt} = \frac{-d}{t^2} + \frac{s \cdot r}{s}$$

Finally, we acquire T:

$$T = \sqrt{\frac{2 \cdot d}{s \cdot r}}$$

Different district can determine their inputs relied on their actual situation, then use our modern to acquire the relative water strategy T. Of course, in different periods, they will get different strategy, such as: seasonal factors will impact the water strategy.

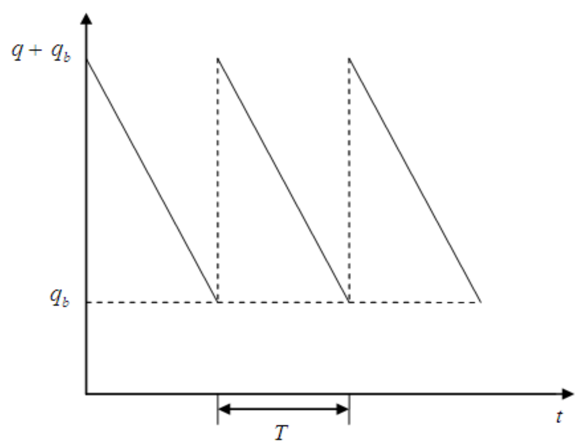

Figure 7: the cycle of q+qb

8 Model Testing based on Monte Carlo Test Model

We have already construct the mathematical model by local dynamic programming, then we need to check the influence of the reservoir stock to the water strategy, which means that the administrator should make sure the sufficient water supply of the buffer stock, in the meantime, make sure the abundant water supply will not cause waste. Thus, we decide to use the Monte Carlo test model to do model testing[23].

Under the idea condition, we assume the demand of water per cubic meter per day(r) is a constant, in reality, r is a variation(Shown in figure 8), thus we use the Monte Carlo test model to analyze the amount of water needs per day (nw).

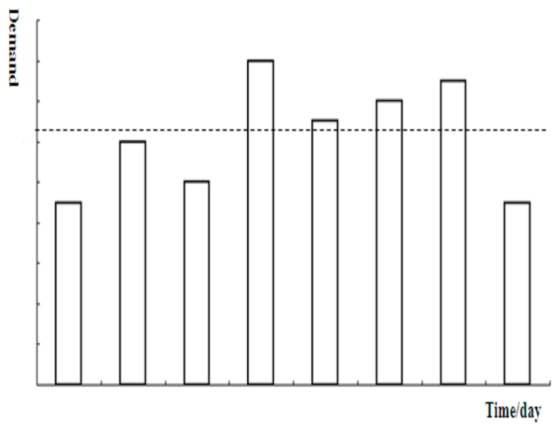

Figure 8: r is a variation

We have already know the water consumption per day in one year, so we can acquire a probability of a range of water(Shown in figure 9) consumption[Figure] , then we accumulate the probability of the range of water consumption successively to get cumulative histogram(Shown in figure 10).

Next we use the middle value to replace each range of water consumption. Using the linear interpolation to connect every point. Finally, we obtain the cumulative function diagram, which means:

$$P = f(nw)$$

Where P is probability of the relative water consumption. It is clearly that we can get the inverse function of f (Shown in figure 11,12), that is:

$$nw = f^{-1}(P)$$

Because the numerical range of P is: $P \subseteq [0, 1]$, thus we can produce random number, which is between 0 and 1, T times to get the amount of water needs every cycle(NW). Compare NW with q+qb to finish our model testing[23].

9 Strengths and Weaknesses

Our models effectively achieve all of the goals we set initially. It is fast and could handle large quantities of data, but also has the flexibility we desired. Though we do

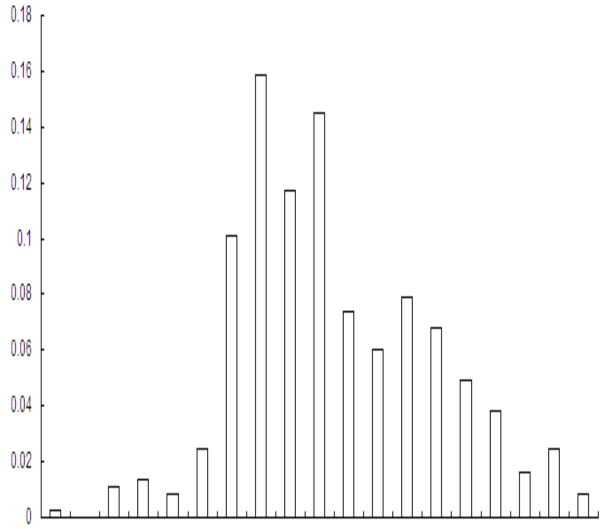

Figure 9: a Range of Water Consumption

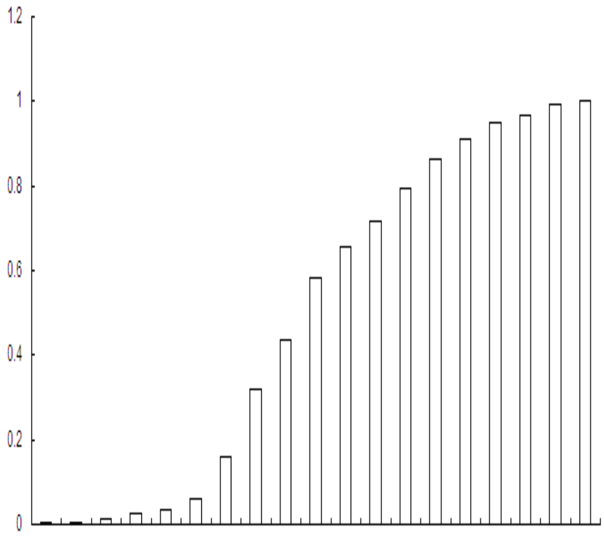

Figure 10: the Cumulative Histogram

not have enough data to test our models, we show that the results of some provinces predicted by the models. As well, we are able to optimize the route of water movement between provinces and obtain the best water strategy which is devoted to the economic impact of water storage and movement, de-salinization and conservation. Our models also consistently lead us to useful minima and help you solve water problem efficiently. Moreover, the models are not limited by the time-and-space restriction.

The primary weaknesses of our models were that we do not have enough data to

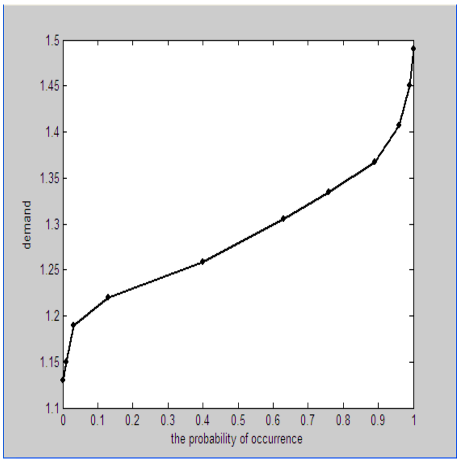

Figure 11: the linear interpolation figure

Random number	Linear spline
$0 \le x < 0.01$	$nw = 2x + 1.13$
$0.01 \le x < 0.03$	$nw = 2x + 1.13$
$0.03 \le x < 0.13$	$nw = 0.3x + 1.18$
$0.13 \le x < 0.40$	$nw = 0.15x + 1.20$
$0.40 \le x < 0.63$	$nw = 0.17x + 1.19$
$0.63 \le x < 0.76$	$nw = 0.31x + 1.11$
$0.76 \le x < 0.89$	$nw = 0.23x + 1.16$
$0.89 \le x < 0.96$	$nw = 0.57x + 0.86$
$0.96 \le x < 0.99$	$nw = 1.33x + 0.13$
$0.99 \le x \le 1$	$nw = 4x - 2.51$

Figure 12: The Function of Linear Interpolation

testify the accuracy of results. Though compactness levels are appropriate measure for comparison, we only divide provinces into three levels in terms of their water shortage degree. Finally, as a more minor weakness, our models make numerous approximations, this weakness is expected in any model, and ours handles such approximations quite well.

10 the non-technical Position Paper to the Government

Our approach is based on the four models, that is, the Grey Prediction model, Fuzzy Mathematical model, Minimum Spanning tree model, and Monte Carlo theory. Each of them can solve the specific problem, for instance, the Grey Prediction model can be used to forecast of water consumption in the future, Fuzzy Mathematical model is served to classify the degrees of the shortage of water at the level of province. Then in our minimum spanning tree model, we consult the information of South-to-North Water Diversion Project. We build the watercourses based on the South-to-North Water Diversion Project, which is the best economical and feasible way, and also we will save lots of money by just building the direct way between two provinces. Secondly, South-to-North Water Diversion Project has already be implemented for many years, which indicates the stability and feasibility of it, thus, our model proposes the water strategy based on the it, will also have the feasibility and the stability. Finally, if people implement their water strategy which acquired by our model, they need only pay attention to the environment, using some environmental-friendly strategy, such as using environmental-friendly tools and so on. Finally, we use Monte Carlo theory to check the feasibility and stability of our model, which guarantee the feasibility once again.

In a nutshell, our approach is feasible, economic, physical and environmental-friendly.

11 Conclusion

Grey prediction model is introduced to forecast the water consumption of each province in China from 2013 to 2025, in the mean time, Fuzzy mathematical model is employed to classify the degrees of water shortage of 31 provinces, the Minimum spanning tree model is utilized to optimize the plan of water transformation and Local dynamic programming(LDP) is used to find the period of delivery of water.

The four models we use solve the problem step by step, and finally through the local dynamic programming, we can get the optimal period of delivery of water, which is our best water strategy, after considering many influential factors.

Our mathematical models is not limited to space limitation, which means people can use our model to predict the water consumption in the future, just using the original sequence, and acquire the best water strategy when they change the inputs according to their demands.

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Appendices

Table 6: the Appendix A.[Unit: a hundred million cubic meters]

	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012
Beijing	34.62	35.0	34.6	34.5	34.3	34.8	35.1	35.5	35.2	35.2
Tianjin	19.96	20.5	22.1	23.1	23.0	23.4	22.3	23.4	22.5	22.5
Hebei	211.38	199.8	195.9	201.8	204.0	202.5	195.0	193.7	193.7	193.7
Shanxi	57.5	56.2	55.9	55.7	59.3	58.7	56.9	56.3	63.8	63.8
Neimenggu	178.23	166.9	171.5	174.8	178.7	180.0	175.8	181.3	181.9	181.9
Liaoning	127.13	128.3	130.2	133.3	141.2	142.9	142.8	142.8	143.7	143.7
Jilin	111.69	104.0	99.2	98.4	102.9	100.8	104.1	111.1	120.0	120.0
Heilongjiang	252.28	245.8	259.4	271.5	286.2	291.4	297.0	316.3	325.0	325.0
Shanghai	104.27	109.0	118.1	121.3	118.6	120.2	119.8	125.2	126.3	126.3
Jiangsu	478.74	433.5	525.6	519.7	546.4	558.3	558.3	549.2	552.2	552.2
Zhejiang	208	206.0	207.8	209.9	208.3	211.0	216.6	197.8	203.0	203.0
Anhui	199.83	178.6	209.7	208.0	241.9	232.1	266.4	291.9	293.1	293.1
Fujian	182.86	182.8	184.9	186.9	187.3	196.3	198.0	201.4	202.5	202.5
Jiangxi	202.06	172.5	203.5	208.1	205.7	234.9	234.2	241.3	239.7	239.7
Shandong	252.37	219.4	214.9	211.0	225.8	219.5	219.9	220.0	222.5	222.5
Henan	218.81	187.6	200.7	197.8	227.0	209.3	227.5	233.7	224.6	224.6
Hubei	240.86	245.1	242.7	253.4	258.8	258.7	270.7	281.4	288.0	288.0
Hunan	306.91	318.8	323.6	328.4	327.7	324.3	323.6	322.3	325.2	325.2
Guangdong	447.03	457.5	464.8	459.0	459.4	462.5	461.5	463.4	469.0	469.0
Guangxi	297.47	278.4	290.8	312.9	314.4	310.4	310.1	303.4	301.6	301.6
Hainan	44.09	46.3	46.3	44.1	46.5	46.7	46.9	44.5	44.4	44.4
Chongqin	60.3	63.2	67.5	71.2	73.2	77.4	82.8	85.3	86.4	86.4
Sichuan	208.61	209.9	210.4	212.3	215.1	214.0	207.6	223.5	230.3	230.3
Guizhou	89.94	93.7	94.3	97.2	100.0	98.0	101.9	100.4	101.4	101.4
Yunnan	148.5	146.1	146.9	146.8	144.8	150.0	153.1	152.6	147.5	147.5
Xizang	30.08	25.3	28.0	33.2	35.0	36.7	37.5	30.9	35.2	35.2
Shanxi	78.01	75.1	75.5	78.8	84.1	81.5	85.5	84.3	83.4	83.4
Gansu	112.64	121.6	121.8	123.0	122.3	122.5	122.2	120.6	121.8	121.8
Qinghai	27.02	29.0	30.2	30.7	32.2	31.1	34.4	28.8	30.8	30.8
Ningxia	81.52	64.0	74.0	78.1	77.6	71.0	74.2	72.2	72.4	72.4
Xinjiang	474.56	500.7	497.1	508.5	513.4	517.7	528.2	530.9	535.1	535.1

Table 7: the Prediction of Water Consumption of Mainland based on GM(1,1) from 2003-2010

	2003	2004	2005	2006	2007	2008	2009	2010
Beijing	34.62	34.5511	34.6406	34.7303	34.8202	34.9103	35.0007	35.0913
Tianjin	19.96	21.9451	22.09	22.2357	22.3825	22.5302	22.6788	22.8285
Hebei	211.38	201.5291	200.5838	199.6428	198.7063	197.7742	196.8464	195.923
Shanxi	57.5	54.9956	55.8427	56.7028	57.5762	58.463	59.3635	60.2778
Neimenggu	178.23	170.3486	171.9693	173.6053	175.257	176.9244	178.6076	180.3068
Liaoning	127.13	130.8446	132.7585	134.7003	136.6706	138.6697	140.6981	142.7561
Jilin	111.69	96.4974	98.9033	101.3692	103.8965	106.4868	109.1417	111.8629
Heilongjiang	252.28	251.9409	260.8838	270.1441	279.7331	289.6625	299.9443	310.5911
Shanghai	104.27	113.8341	115.4538	117.0964	118.7624	120.4521	122.1659	123.9041
Jiangsu	478.74	493.484	502.8957	512.4868	522.2609	532.2215	542.3719	552.716
Zhejiang	208	209.8469	209.1407	208.437	207.7356	207.0366	206.3399	205.6456
Anhui	199.83	191.4091	203.2282	215.7772	229.1011	243.2477	258.2678	274.2154
Fujian	182.86	182.4275	185.1299	187.8724	190.6555	193.4799	196.3461	199.2547
Jiangxi	202.06	190.6929	197.4324	204.4101	211.6344	219.114	226.858	234.8757
Shandong	252.37	216.3582	217.137	217.9186	218.703	219.4903	220.2804	221.0733
Henan	218.81	196.2284	200.6246	205.1193	209.7147	214.4131	219.2167	224.1279
Hubei	240.86	240.7826	246.5488	252.4531	258.4988	264.6892	271.0279	277.5184
Hunan	306.91	323.4083	323.6419	323.8757	324.1097	324.3439	324.5782	324.8127
Guangdong	447.03	458.2692	459.4201	460.5738	461.7305	462.89	464.0525	465.2178
Guangxi	297.47	296.0402	297.6649	299.2985	300.9412	302.5928	304.2535	305.9233
Hainan	44.09	46.3719	46.1685	45.966	45.7644	45.5637	45.3639	45.1649
Chongqin	60.3	65.3407	67.9995	70.7665	73.6461	76.6428	79.7615	83.0071
Sichuan	208.61	206.655	209.1779	211.7315	214.3163	216.9326	219.5809	222.2615
Guizhou	89.94	94.7527	95.7162	96.6895	97.6728	98.666	99.6694	100.6829
Yunnan	148.5	146.5652	147.0124	147.4609	147.9108	148.362	148.8147	149.2687
Xizang	30.08	29.4151	30.2553	31.1194	32.0083	32.9225	33.8629	34.8301
Shanxi	78.01	76.8288	77.9079	79.0021	80.1117	81.2369	82.3779	83.5349
Gansu	112.64	122.2286	122.1603	122.092	122.0237	121.9554	121.8872	121.8191
Qinghai	27.02	30.4091	30.528	30.6473	30.7671	30.8874	31.0081	31.1293
Ningxia	81.52	71.9987	72.217	72.4358	72.6554	72.8756	73.0965	73.3181
Xinjiang	474.56	498.0507	503.0488	508.0971	513.196	518.3461	523.5479	528.8018

Table 8: the Prediction of Water Consumption of Mainland based on GM(1,1) from 2011-2018

	2011	2012	2013	2014	2015	2016	2017	2018
Beijing	35.1821	35.2732	35.3645	35.4561	35.5479	35.6399	35.7322	35.8247
Tianjin	22.9791	23.1308	23.2834	23.437	23.5917	23.7474	23.9041	24.0618
Hebei	195.0039	194.0892	193.1787	192.2725	191.3705	190.4728	189.5793	188.69
Shanxi	61.2063	62.149	63.1062	64.0782	65.0652	66.0674	67.085	68.1183
Neimenggu	182.0222	183.754	185.5022	187.267	189.0486	190.8472	192.6629	194.4959
Liaoning	144.8442	146.9629	149.1125	151.2936	153.5066	155.752	158.0302	160.3417
Jilin	114.6518	117.5103	120.4401	123.4429	126.5205	129.6749	132.908	136.2216
Heilongjiang	321.6158	333.0319	344.8531	357.094	369.7694	382.8947	396.4859	410.5595
Shanghai	125.6669	127.4549	129.2683	131.1075	132.9729	134.8648	136.7836	138.7297
Jiangsu	563.2573	573.9997	584.947	596.103	607.4718	619.0575	630.8641	642.8959
Zhejiang	204.9536	204.2639	203.5765	202.8915	202.2088	201.5284	200.8502	200.1743
Anhui	291.1477	309.1255	328.2135	348.4801	369.9981	392.8448	417.1023	442.8576
Fujian	202.2065	205.202	208.2418	211.3267	214.4572	217.6342	220.8582	224.13
Jiangxi	243.1767	251.7711	260.6692	269.8819	279.4201	289.2954	299.5198	310.1055
Shandong	221.8691	222.6678	223.4693	224.2737	225.081	225.8913	226.7044	227.5205
Henan	229.1492	234.2829	239.5317	244.8981	250.3847	255.9942	261.7294	267.593
Hubei	284.1644	290.9694	297.9375	305.0724	312.3782	319.859	327.5188	335.3622
Hunan	325.0473	325.2822	325.5172	325.7523	325.9877	326.2232	326.4589	326.6947
Guangdong	466.3861	467.5574	468.7316	469.9087	471.0888	472.2718	473.4578	474.6468
Guangxi	307.6023	309.2905	310.988	312.6947	314.4109	316.1364	317.8715	319.616
Hainan	44.9668	44.7696	44.5733	44.3778	44.1831	43.9894	43.7964	43.6044
Chongqin	86.3848	89.8999	93.5581	97.3651	101.327	105.4502	109.7411	114.2066
Sichuan	224.9748	227.7213	230.5013	233.3152	236.1635	239.0466	241.9648	244.9187
Guizhou	101.7068	102.741	103.7858	104.8412	105.9073	106.9843	108.0722	109.1712
Yunnan	149.7241	150.1809	150.639	151.0986	151.5596	152.022	152.4858	152.951
Xizang	35.8249	36.8482	37.9006	38.9832	40.0966	41.2419	42.4198	43.6315
Shanxi	84.7082	85.8979	87.1044	88.3278	89.5684	90.8264	92.1021	93.3956
Gansu	121.7509	121.6828	121.6148	121.5468	121.4788	121.4108	121.3429	121.2751
Qinghai	31.251	31.3732	31.4958	31.6189	31.7425	31.8666	31.9912	32.1162
Ningxia	73.5403	73.7632	73.9868	74.211	74.436	74.6616	74.8879	75.1149
Xinjiang	534.1085	539.4685	544.8822	550.3503	555.8733	561.4516	567.086	572.7769

Table 9: the Prediction of Water Consumption of Mainland based on GM(1,1) from 2019-2025

	2019	2020	2021	2022	2023	2024	2025
Beijing	35.9174	36.0104	36.1036	36.1971	36.2908	36.3848	36.4789
Tianjin	24.2206	24.3804	24.5413	24.7033	24.8663	25.0304	25.1955
Hebei	187.8049	186.9239	186.047	185.1743	184.3056	183.441	182.5805
Shanxi	69.1675	70.2328	71.3146	72.413	73.5284	74.6609	75.8108
Neimenggu	196.3463	198.2143	200.1001	202.0038	203.9256	205.8657	207.8243
Liaoning	162.6871	165.0667	167.4812	169.931	172.4166	174.9386	177.4974
Jilin	139.6179	143.0989	146.6666	150.3233	154.0711	157.9124	161.8495
Heilongjiang	425.1327	440.2232	455.8493	472.0301	488.7853	506.1352	524.1009
Shanghai	140.7035	142.7054	144.7358	146.7951	148.8836	151.0019	153.1504
Jiangsu	655.1571	667.6522	680.3856	693.3618	706.5856	720.0615	733.7944
Zhejiang	199.5008	198.8294	198.1604	197.4936	196.829	196.1667	195.5066
Anhui	470.2033	499.2375	530.0645	562.7951	597.5467	634.4441	673.6199
Fujian	227.4502	230.8196	234.239	237.709	241.2304	244.8039	248.4304
Jiangxi	321.0653	332.4124	344.1606	356.324	368.9173	381.9557	395.4548
Shandong	228.3395	229.1614	229.9863	230.8142	231.645	232.4789	233.3157
Henan	273.5881	279.7174	285.9841	292.3911	298.9417	305.6391	312.4865
Hubei	343.3933	351.6168	360.0372	368.6593	377.4878	386.5278	395.7843
Hunan	326.9307	327.1669	327.4033	327.6398	327.8765	328.1134	328.3504
Guangdong	475.8388	477.0338	478.2318	479.4328	480.6368	481.8438	483.0539
Guangxi	321.3702	323.1339	324.9074	326.6905	328.4835	330.2863	332.099
Hainan	43.4131	43.2227	43.0331	42.8444	42.6565	42.4694	42.2832
Chongqin	118.8538	123.6901	128.7233	133.9612	139.4123	145.0851	150.9889
Sichuan	247.9086	250.9351	253.9984	257.0992	260.2379	263.4148	266.6305
Guizhou	110.2814	111.4029	112.5357	113.6801	114.8361	116.0039	117.1835
Yunnan	153.4176	153.8857	154.3552	154.8261	155.2985	155.7723	156.2475
Xizang	44.8777	46.1595	47.4779	48.834	50.2288	51.6635	53.1391
Shanxi	94.7074	96.0376	97.3865	98.7543	100.1413	101.5478	102.9741
Gansu	121.2073	121.1395	121.0717	121.004	120.9363	120.8687	120.8011
Qinghai	32.2418	32.3678	32.4943	32.6213	32.7489	32.8769	33.0054
Ningxia	75.3426	75.5709	75.8	76.0297	76.2602	76.4913	76.7232
Xinjiang	578.5249	584.3306	590.1945	596.1173	602.0995	608.1418	614.2447

Table 10: the Accuracy Evaluation of Water Consumption of 30 Provinces

	Mean value of ralative error	Absolute correlation degree	Mean square error ratio
Beijing	0.17428	0.9935	0.7493
Tianjing	0.1633	0.993	0.64
Hebei	0.15395	0.9991	0.5404
Shanxi	0.16027	0.9447	0.658
Neimenggu	0.14327	0.9679	0.4648
Liaoning	0.13903	0.9916	0.3987
Jilin	0.15116	0.9892	0.5224
Heilongjiang	0.11554	0.9935	0.1619
Shanghai	0.13464	0.9978	0.3486
Jiangsu	0.0381	0.9882	0.6429
Zhejiang	0.0156	0.9563	**0.9339**
Anhui	0.0393	0.9901	0.2698
Fujian	0.0075	0.9921	0.2365
Jiangxi	0.0372	0.9801	0.4368
Shandong	0.0099	0.9998	0.3188
Henan	0.0325	0.9466	0.5842
Hubei	0.01	0.9958	0.1907
Hunan	0.0051	0.9999	0.4275
Guangdong	0.0041	0.9973	0.3935
Shanxi	0.0271	0.9572	**0.9094**
Hainan	0.0156	0.9932	**0.8137**
Chongqin	0.0167	0.9938	0.1944
Sichuan	0.0139	0.9907	0.5457
Guizhou	0.0103	0.9954	0.3334
Yunnan	0.0124	0.7361	**0.9031**
Xizang	0.0783	0.9848	0.7515
Shanxi	0.0199	0.9773	0.5728
Gansu	0.0036	0.9996	0.2017
Qinhai	0.0325	0.9957	0.7754
Ningxia	0.0377	0.9954	**0.8088**
Xiangjiang	0.0042	0.997	0.161

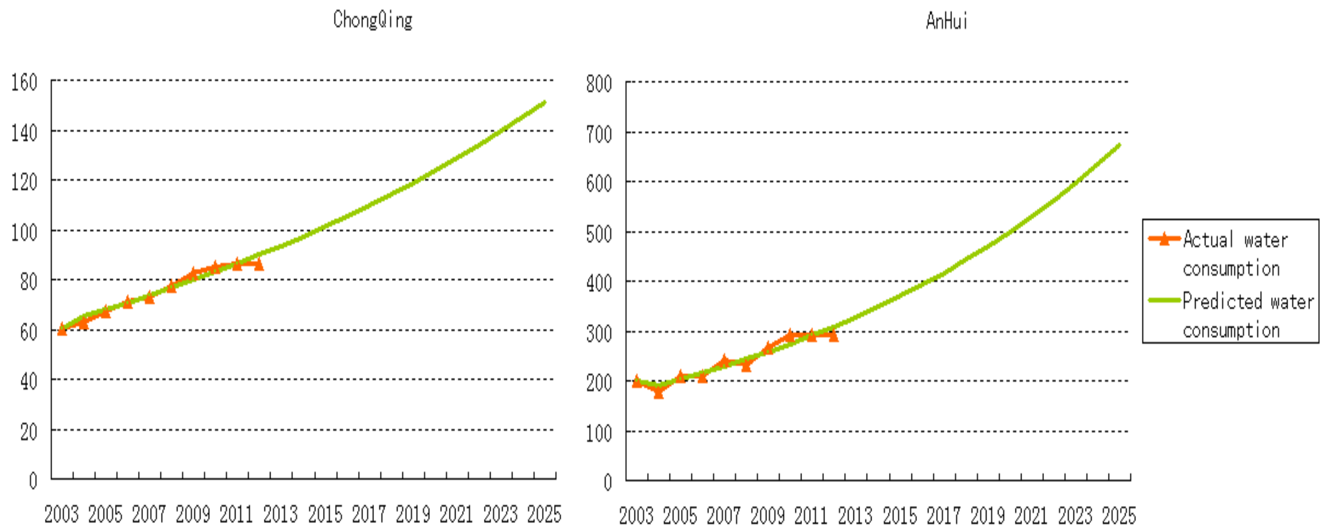

Figure 13: the Water Consumption of Chongqing and Anhui

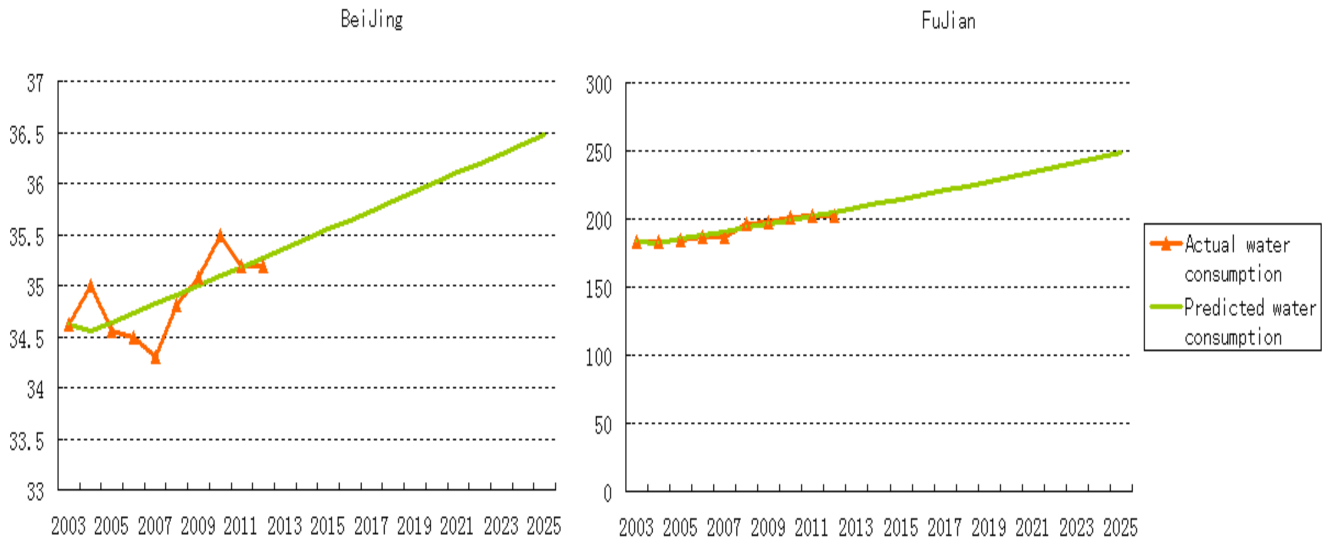

Figure 14: the Water Consumption of Beijing and Fujian

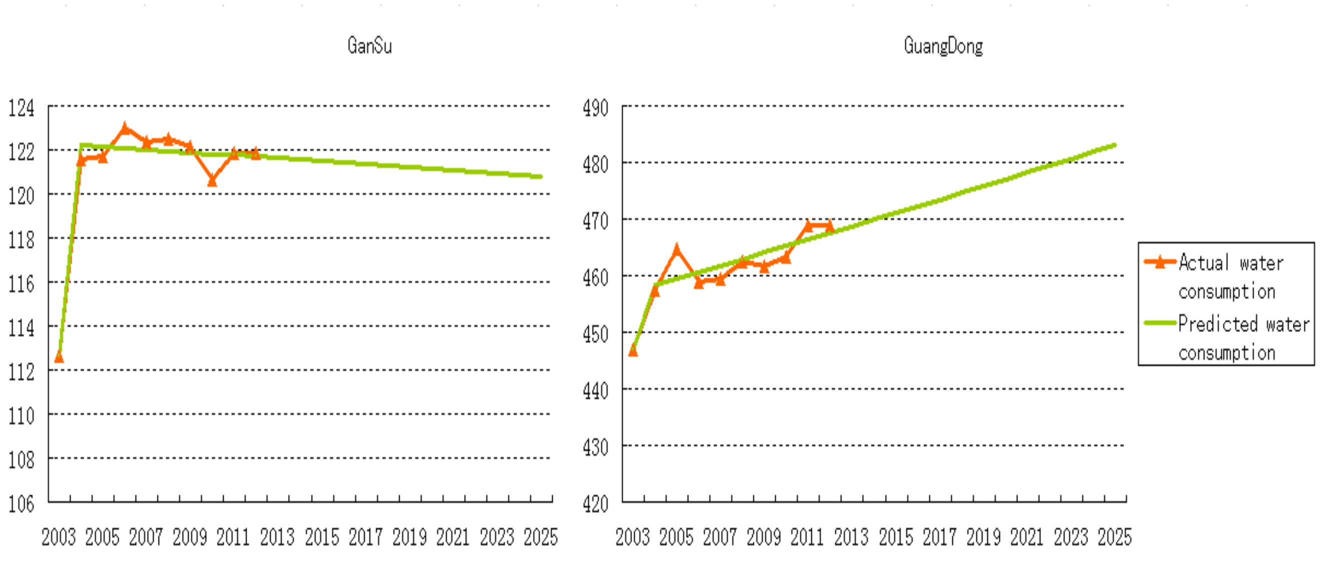

Figure 15: the Water Consumption of Gansu and Guangdong

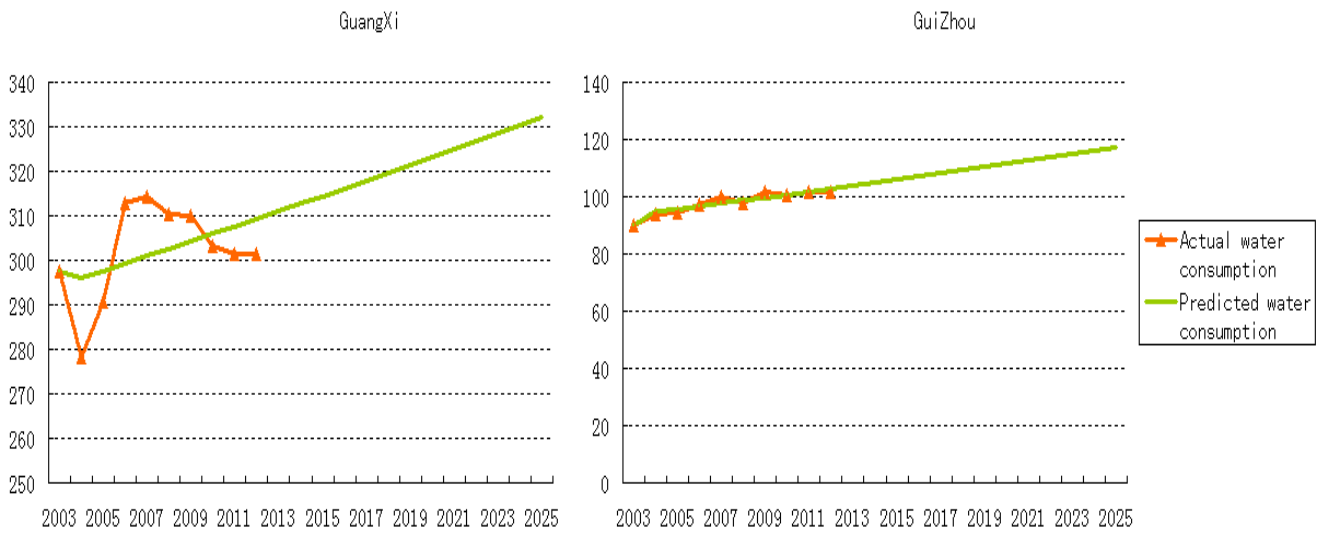

Figure 16: the Water Consumption of Guangxi and Guizhou

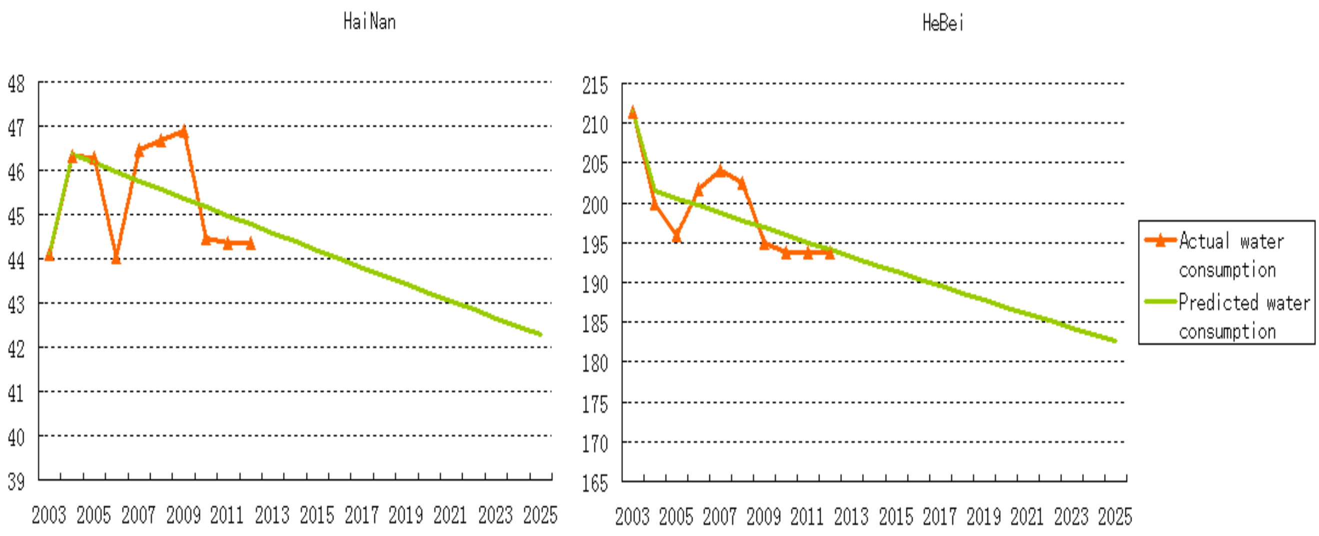

Figure 17: the Water Consumption of Hainan and Hebei

Figure 18: the Water Consumption of Henan and Heilongjiang

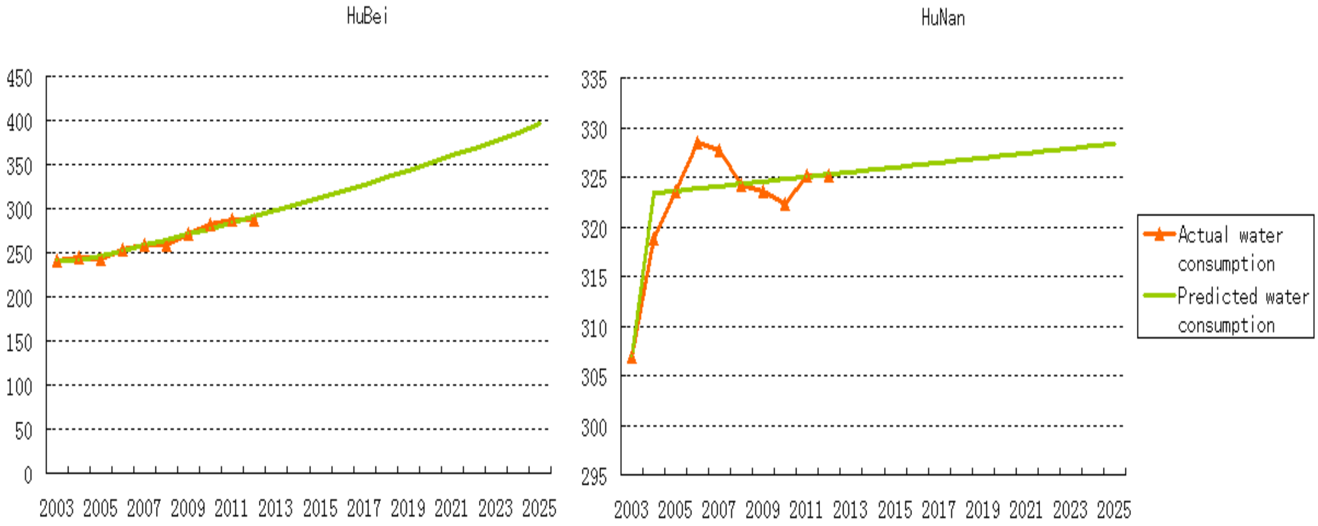

Figure 19: the Water Consumption of Hubei and Hunan

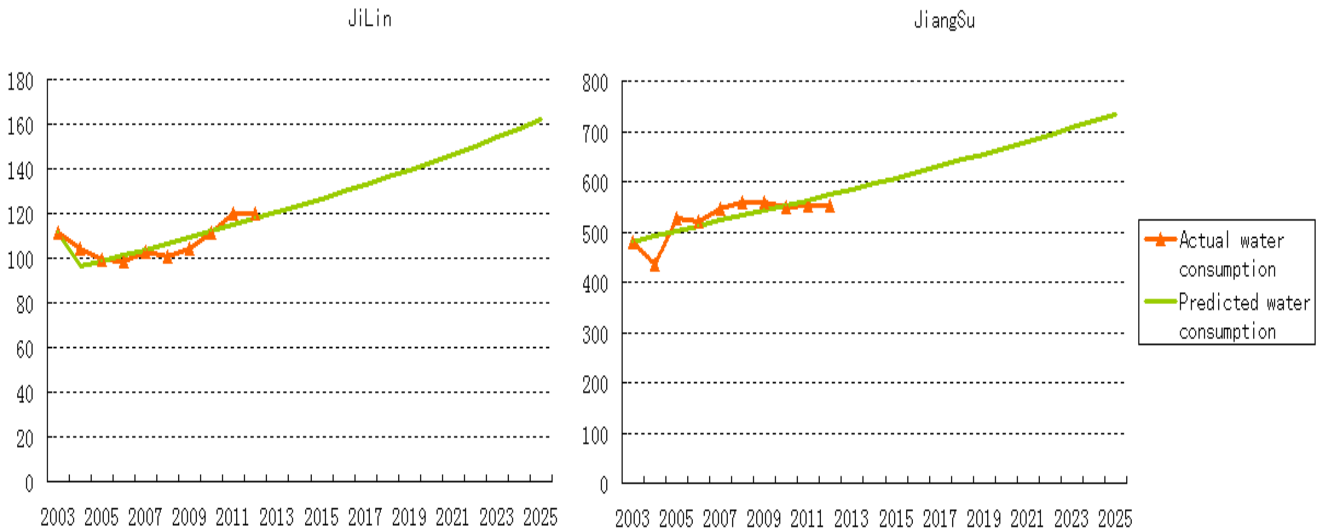

Figure 20: the Water Consumption of Jilin and Jiangsu

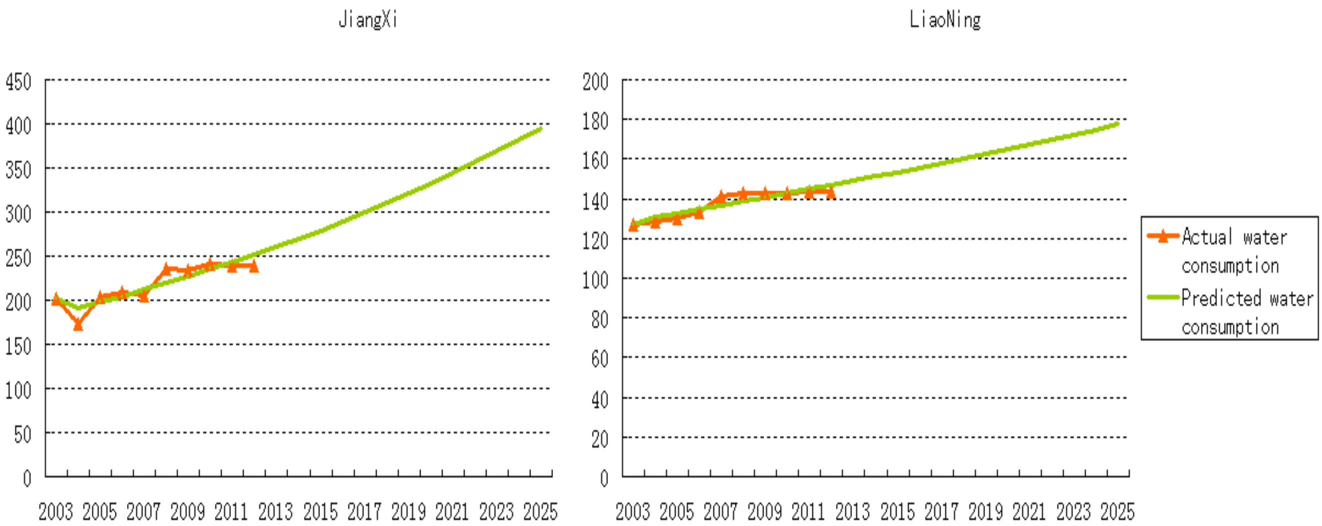

Figure 21: the Water Consumption of Jiangxi and Liaoning

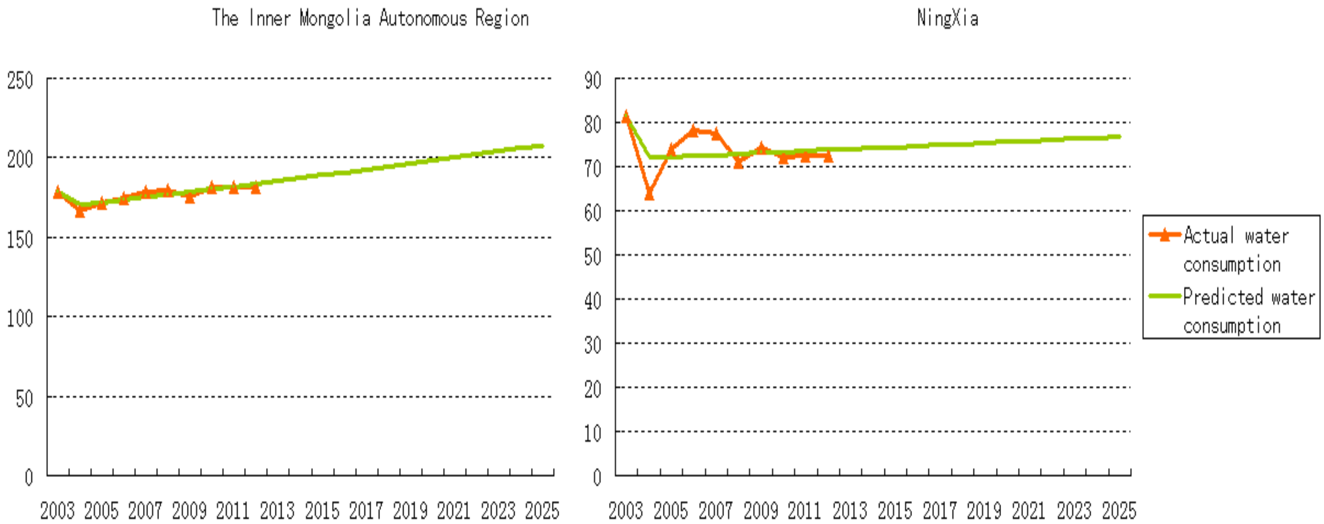

Figure 22: the Water Consumption of Neimenggu and Ningxia

Figure 23: the Water Consumption of Qinghai and Shandong

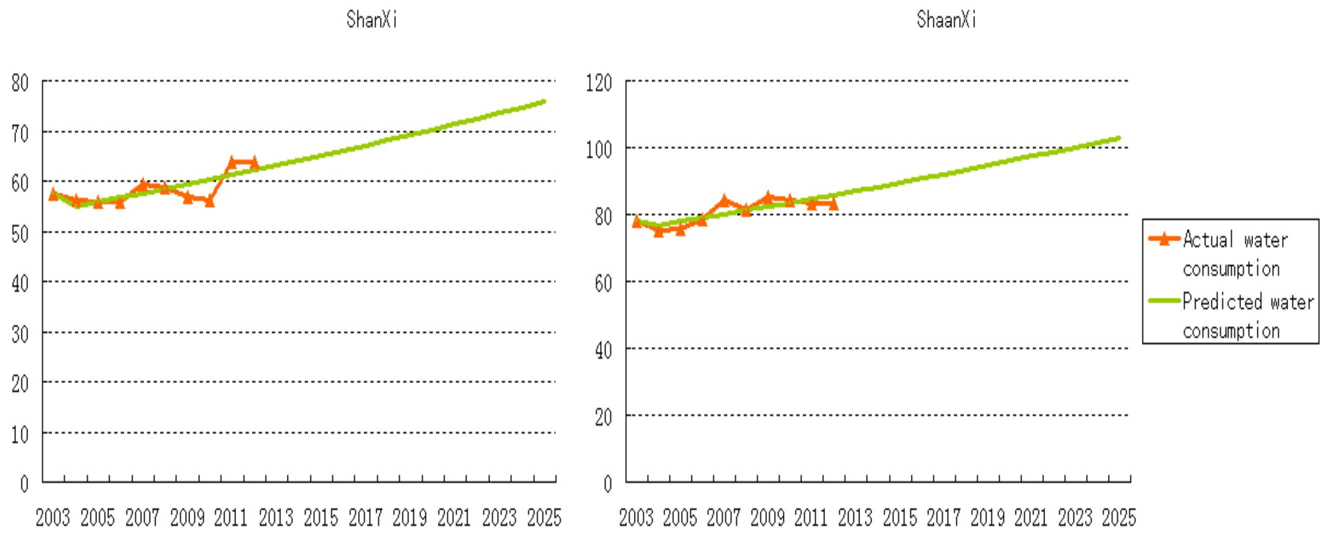

Figure 24: the Water Consumption of Shanxi and Shannxi

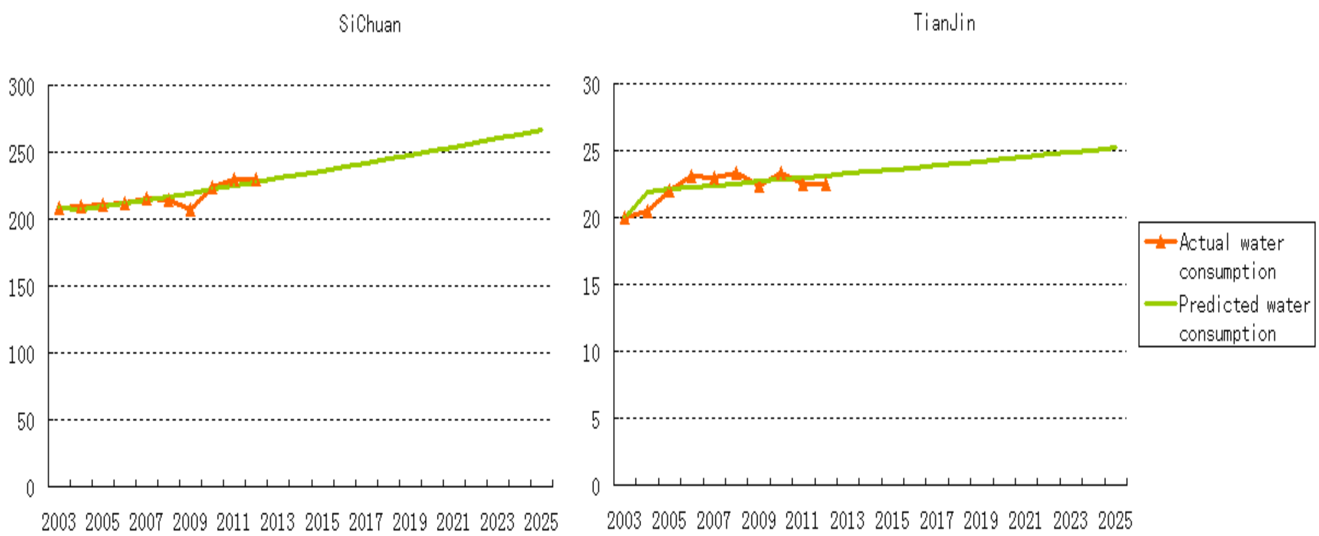

Figure 25: the Water Consumption of Sichuan and Tianjin